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Magnetic focusing of composite fermions in double quantum wells

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Abstract. In this work, we have numerically integrated in space and time the effective-mass nonlinear Schrödinger equation for a composite fermion in a double-quantum-well system. Considering many-body effects and the existence of an external applied bias, a time-varying composite-fermion effective magnetic field with an amplitude which is oscillating with time in each quantum well has been obtained. As a consequence, the magnetic focusing peak positions in the magnetoresistance measurements will be shifted to a lower magnetic field value.

1. Introduction

Since the discovery of the integer and fractional quantum Hall effects (QHE), the properties of two-dimensional (2D) electron systems in a strong magnetic field have been the focus of great attention [1]. In the fractional quantum Hall effect, the composite-fermion (CF) model has provided us with an intuitive picture for interpreting electron correlation phenomena at high magnetic fields [2, 3]. The CF model postulates the existence of new particles called composite fermions, which consist of electrons to which an even number ($\phi = 2m$) of magnetic flux quanta have been attached by virtue of many-body interactions. Hence, we are able to obtain a mapping of the fractional QHE at Landau level filling factor $\nu = p/(2mp+1)$, where m and p are integers, onto the integer QHE at $\nu^* = |p|$, where ν^* is the CF Landau level filling factor.

Lopez and Fradkin [4] showed formally the equivalence between a system of electrons at filling factors ν and a system of fermions interacting with a Chern–Simons gauge field at integer filling factor p. The fermion Chern–Simons picture of Lopez and Fradkin was further developed by Halperin, Lee and Read [3], who used it to study the case of a $\nu = 1/2$ filling fraction. Furthermore, the existence of the CFs at filling 1/2 has been experimentally checked by (i) studying the cyclotron resonance of the CFs near $\nu = 1/2$ [5], (ii) studying the cyclotron resonance of the CFs in an array of antidots [6], (iii) observing that the gaps increase linearly with the magnetic field [7], (iv) studying the Shubnikov–de Haas oscillations at around $\nu = 1/2$ [8] and (v) detecting the CFs by magnetic focusing [9, 10].

In addition to this, and in the fractional QHE regime, the tunnelling between two parallel 2D electron systems has been recently studied by Eisenstein, Pfeiffer and West [11, 12]. In this experiment, the I–V characteristics exhibited a strong suppression of the tunnelling current at low biases and a pronounced peak in the neighbourhood of $eV_{\max} \sim e^2/4\pi\varepsilon\langle a\rangle$, where ε is the dielectric constant and $\langle a\rangle$ is the average inter-electron spacing. This result suggested that there is an excitonic energy cost of $eV_{\max}$ for moving an electron from one

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quantum well to the other. In contrast, in the case of $B = 0$ the sample exhibited a sharply peaked conductance at zero bias.

From a theoretical point of view, the experimental suppression of the tunnelling current in symmetric quantum wells [11, 12] has been analysed using a classical model for the description of some properties of a 2D electron system in a strong magnetic field [13]. In addition to this, such an effect has also been studied using a Wigner crystal model [14], assuming a hydrodynamic model for low-energy excitations in a quantum liquid [15], with the proposal of an exact sum rule for the tunnelling density of states [16], and calculating the one-electron Green's functions in an interacting 2D electron system [17].

In the CF model, the detection of CFs by magnetic focusing was experimentally reported by Goldman *et al* [9]. They observed quasiperiodic magnetic focusing peaks in heterojunction samples. The direction of focusing was in good quantitative agreement with that expected from semiclassical transport by CFs of charge e. They showed that the classical focusing occurs up to an effective magnetic field for which the cyclotron radius is approximately equal to the undepleted opening in the constriction. The samples were prepared from a high-mobility GaAs–GaAlAs heterojunction. However, we can notice that a double-quantum-well semiconductor heterostructure, originally designed for studying the tunnelling between two parallel 2D electron systems [11, 12], could be chosen in these magnetic focusing experiments. In such a case, we can expect modification of the classical focusing condition due to the tunnelling oscillations between the quantum wells. In this way, the experimental suppression of the tunnelling current in a correlated quantum liquid could be studied using magnetoresistance measurements.

In view of the above comments, and from a theoretical point of view, we can see that there is an urgent need to evaluate the CF dynamics in a double-quantum-well system. In this work, we will propose a one-dimensional effective-mass Schrödinger equation for studying the charge-density dynamics in the heterostructure growth direction (along the z-axis). In the semiconductor layer plane (the x–y plane), the time-dependent CF dynamics will also be analysed. Many-body effects and a time-dependent CF effective magnetic field will be considered in our model. Finally, we shall show that the tunnelling oscillations between the quantum wells will allow the existence of a new kind of CF dynamics in the x–y plane, and thus a new kind of magnetoresistance measurement for the system.

2. The model

In order to study the dynamics of CFs in the structure growth direction, we need to solve the time-dependent Schrödinger equation associated with a CF in a quantum well potential. The CF wave function ψ_{CF} will be given by the nonlinear Schrödinger equation

$$\mathrm{i}\hbar \frac{\partial}{\partial t}\psi_{\mathrm{CF}}(z,t) = \left[-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2} + V_{\mathrm{e-e}}(|\psi_{\mathrm{CF}}|^2) + V_{\mathrm{qw}}(z) + V_{\mathrm{bias}}(z) + V_{\mathrm{exc}}\right]\psi_{\mathrm{CF}}(z,t) \tag{1}$$

where m^* is the electron GaAs effective mass in the growth direction, V_{qw} is the quantum well potential, V_{bias} is a potential due to an external applied bias, V_{exc} is an excitonic potential due to correlation effects and $V_{\mathrm{e-e}}$ is the potential given by the electron–electron interaction in the heterostructure region. Such a many-body potential is given by Poisson's equation

$$\frac{\partial^2}{\partial z^2}V_{\mathrm{e-e}}(z,t) = \frac{en_s}{\varepsilon}|\psi_{\mathrm{CF}}(z,t)|^2 \tag{2}$$

where n_s is the initial electronic sheet density. We should point out that in equation (1) the $V_{\mathrm{e-e}}$ many-body potential is a time-dependent quantity. Such a result is given by equation (2) where V_{e-e} depends on the wave-function form.

Now we study the form of the excitonic potential in equation (1). The V_{exc}-term accounts for electron correlation effects. In principle, it is possible to obtain a strongly correlated system in a GaAs semiconductor layer if we apply a magnetic field large enough to force the Fermi level into the lowest Landau level. Under these conditions, an electron that tunnels between the two layers will produce a strongly localized charge defect [11, 12]. For example, if an electron tunnels from the left-hand quantum well to the right-hand semiconductor layer, due to an external bias, an excitonic $-q$-charge will be produced in the right-hand quantum well. In addition, the hole that the electron that has tunnelled leaves behind in the left-hand quantum well will also produce a $+q$-charge in the left-hand semiconductor layer. Then, we have opposite excitonic charges $+q$ and $-q$ in the two quantum wells [18].

In order to obtain an equation for V_{exc}, we will consider the two semiconductor layers as two planes of area A bearing equal but opposite excitonic charges $-q$ and $+q$ [18]. Then, the potential difference can be written as

$$V_{exc} = 4\pi k \frac{q}{A} d \tag{3}$$

where d is the distance between the planes and k the Coulomb constant for GaAs. In our case, q/A is the charge density of electrons that have tunnelled (or excitonic holes) in each quantum well and d is the quantum well centre-to-centre spacing. The V_{exc}-term accounts for correlation effects. In the absence of an external applied magnetic field, we have that $V_{exc} = 0$ in equation (1). In the case of a strongly correlated 2D electron system, V_{exc} is given by equation (3), i.e., $V_{exc} \neq 0$ in equation (1). In the experiments [11, 12], correlation effects were found over a broad range of Landau level filling fraction ($0.48 < \nu < 0.83$).

In addition, we know that the numerical integration over time of equation (1) allows us to obtain the electron charge density Q_{ab} in a defined semiconductor region $[a, b]$ at any time t:

$$Q_{ab}(t) = \int_a^b \mathrm{d}z\ |\psi(z,t)|^2 \tag{4}$$

where a and b are the quantum well limits. Assuming that $Q_{left} = Q_{right}$ at $t = 0$, the charge density of electrons that have tunnelled (or excitonic holes) in a quantum well at $t > 0$ can be written as $q/A = (Q_{ab}(t) - Q_{ab}(t = 0))n_s/2$, i.e.,

$$\frac{q}{A} = \frac{n_s}{2}\left(\int_a^b \mathrm{d}z\ |\psi(z,t)|^2 - \int_a^b \mathrm{d}z\ |\psi(z,t=0)|^2\right) \tag{5}$$

where n_s is the initial 2D electron sheet density in both quantum wells. At this point, we can notice that in equation (5) the charge density of electrons that have tunnelled (or excitonic holes) in a quantum well is a time-dependent quantity. If the sign of q/A is positive, we have a sheet density of electrons that have tunnelled in the quantum well. In contrast, if the sign is negative, we can find a hole charge density in the semiconductor layer.

In the CF model, part of the external magnetic field is incorporated into the new particle and, then, the CF experiences a reduced (effective) magnetic field

$$B^* = B - \phi\, 2\pi n_s \frac{\hbar c}{e} \tag{6}$$

where n is the electron (and CF) density in each quantum well. In this way, the CFs experience zero net magnetic field at exactly half-filling, so they are expected to form a Fermi surface and have a renormalized mass, dependent only on many-body interactions. As the applied magnetic field B deviates from the average magnetic field due to the composite fermions $B_{\rm av} = \phi\, 2\pi n_s \hbar c/e$, the effective magnetic field B^* quantizes the CFs into Landau levels in analogy to Landau quantization of usual electrons.

In the double-quantum-well case, we can define a time-dependent effective magnetic field in each quantum well as

$$B^*_{L\,(R)}(t) = B - \phi\, 2\pi n_s \frac{\hbar c}{e} Q_{L\,(R)}(t) \tag{7}$$

where $Q_L(t)$ and $Q_R(t)$ are the charge densities in the left-hand and right-hand quantum wells, respectively. Taking into account equation (4), both of the effective magnetic fields, B^*_L and B^*_R, can be easily calculated.

Now we discretize time using a superscript n and spatial position using a subscript r. Thus, $\psi_{\rm CF} \rightarrow \kappa^n_r$. The various z-values become $r\,\delta z$ in the conduction band, where δz is the mesh width. Similarly, the time variable takes the values $n\,\delta t$, where δt is the time step. In this way, and to treat the time development, we have used a unitary propagation scheme for the evolution operator, obtaining a tridiagonal linear system that can be solved by standard numerical methods [19]. In addition, we have also solved the Poisson equation associated with $V_{\rm e-e}$ using another standard tridiagonal numerical method for each t-value. Equations (1) and (2) are both self-consistently solved.

In our calculations, we have taken two different Gaussian wave packets centred in the two quantum wells as our initial wave function. A small part of the wave function is localized in the quantum well barrier regions. Taking into account the definition of q/A in equation (5), we can notice that there is no contribution from such small wave-packet parts to the calculated electron density.

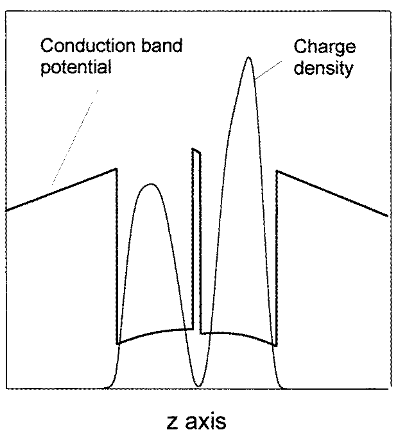

Figure 1. The conduction band potential and wave function at $t = 2$ ps. We have taken $V_{\rm bias} = 14$ mV.

3. Results and discussion

In figure 1, the amplitude of the wave function $|\psi|^2$ and the conduction band potential at $t = 2$ ps are shown. In this figure, we have taken $V_{\rm bias} = 14$ mV. We have numerically integrated equations (1) and (2) using an initial 2D electron sheet density equal to $n_s \sim 1.6 \times 10^{11}$ cm^{-2} for each quantum well. Then, the equations were solved numerically using a spatial mesh size of 0.5 Å and a time mesh size of 0.2 au, and a finite box (4000 Å) large enough for us to be able to neglect border effects. We have considered a GaAs/$Ga_{0.7}Al_{0.3}$As double-quantum-well system which consists of a left-hand quantum well 200 Å wide, a right-hand quantum well 200 Å wide and a barrier of thickness equal to 20 Å.

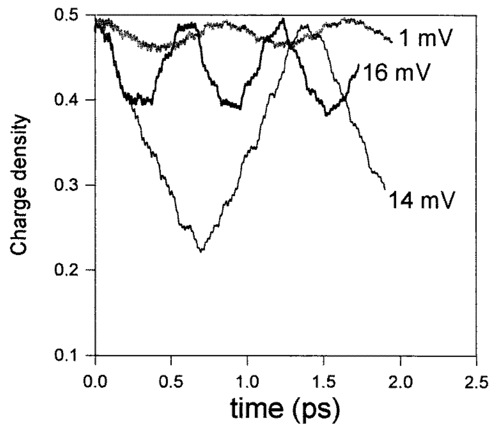

Figure 2. The charge density in the left-hand quantum well (Q_{ab}) versus time at different applied biases.

In figure 2, we have plotted the charge density in the left-hand quantum well versus time at different applied biases. The charge-density values have been obtained through the use of equation (4). The total charge density in both quantum wells has been taken as equal to 1. In figure 2, the existence of tunnelling oscillations between the two quantum wells is clearly shown. We can also notice that smooth curves have not been obtained from our numerical integration. The different lines shown in figure 2 display small oscillations. Such a result is given by equation (1) where V_{e-e} depends on the wave-function form in our nonlinear Schrödinger equation. Due to the V_{e-e} potential being a wave-dependent quantity, the charge dynamically trapped in the two wells produces a reaction field which modifies the time evolution of the system. As a result, the charge density localized in the left-hand quantum well displays small oscillations as time progresses.

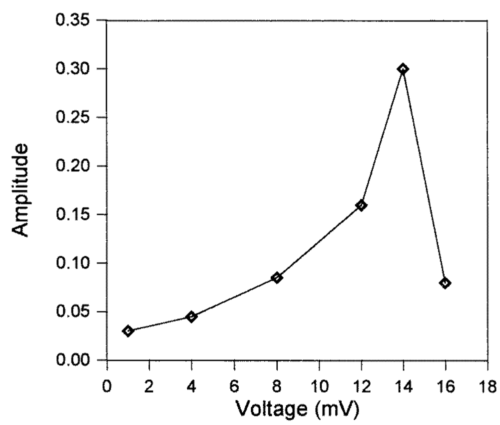

Figure 3. The amplitude of the tunnelling oscillations versus the applied bias.

In figure 3, we have plotted the amplitude of the tunnelling oscillations versus applied bias. Up to a certain value of the applied bias ($V_{bias} = 14$ mV), we have found that the amplitude of the tunnelling oscillations is increased as we increase V_{bias}. In contrast, and in the case of $V_{\text{bias}} > 14$ mV, it is found that the amplitude is decreased if we increase V_{bias}. Such a result can be easily explained, if we consider two new points. Firstly, we know that the electron energy levels of the two wells are exactly aligned at $V_{\text{bias}} = 0$. On increasing V, the amplitude of the oscillations will also be increased due to the field-induced tunnelling process (figure 2). However, we know that if the applied bias is higher than the level splitting between the two quantum wells, the resonant condition is not obtained and, then, the tunnelling process is not allowed. Secondly, and from equations (3) and (5), we know that if some electrons tunnel from one quantum well to another under the action of an external bias, we have a finite value for V_{exc}. Then, the excitonic field obtained is in opposition to the applied bias and the process of tunnelling between the two wells is slowed. In this way, the tunnelling peak is shifted to a higher voltage value. In other words, there is an excitonic energy cost for moving an electron from one quantum well to the other due to the field-induced V_{exc}-term. Taking into account the relation between the oscillation amplitude and the tunnelling current, we can see that both effects are consistent with available experimental data. In experiments, Eisenstein *et al* [11] found a suppression of the tunnelling current at low biases and a pronounced peak at a higher voltage value.

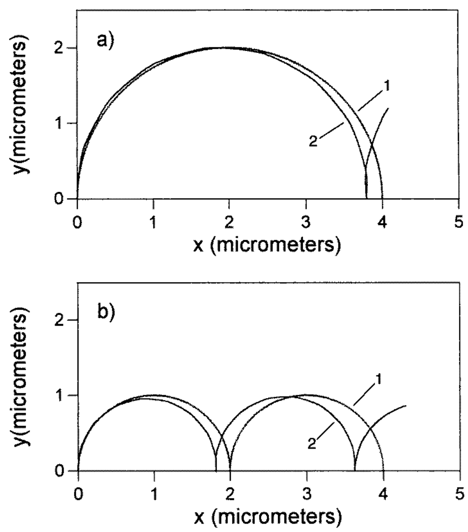

Figure 4. Composite-fermion trajectories. (a) The $j = 1$ case. (b) The $j = 2$ case. Line 1: B^* is constant. Line 2: $m^*_{\text{CF}} = 0.067m_0$ and $V_{\text{bias}} = 0.1$ mV.

For the single-quantum-well case, Goldman *et al* [9] reported the observation of CFs in transverse-focusing experiments. They observed quasiperiodic resistance peaks in the focusing geometry as expected for CFs. The sample geometry used in the experiment is shown in figure 4. The current is passed through the left-hand constriction ($x = 0$ μm in figure 4) and the voltage developed across the right-hand constriction ($x = 4$ μm in figure 4) is measured as a function of the magnetic field. From a semiclassical point of view, CFs coming out of the left-hand constriction with the Fermi velocity execute cyclotron motion. When B^* is such that $2jR = l$ (l is the constriction separation, j is an integer and R is the cyclotron radius), the CFs are focused into the right-hand constriction. We have assumed specular reflections from flat gates. In figure 4(a) and figure 4(b), we have plotted the $j = 1$ and $j = 2$ cases (lines 1), respectively. Then, focusing peaks occur at $B^*_j = j\,\Delta B^*$, with the period $\Delta B^* = 2\hbar k^*_F/el$ where k^*_F is the Fermi wave vector of the CFs.

In the double-quantum-well case, we have charge-density oscillations between the two quantum wells (figure 2). As a consequence, and from equation (7), the amplitude of the effective magnetic field is also oscillating with time. The motion of the CF wave packets in each quantum well can be described by the semiclassical equations of motion [20]

$$\hbar \frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{k}_F^* = -e(\boldsymbol{v} \times \boldsymbol{B}_{L\,(R)}^*) \tag{8}$$

where $\boldsymbol{v} = (1/\hbar)\partial\varepsilon_{\mathrm{CF}}(\boldsymbol{k})/\partial\boldsymbol{k}$ for a dispersion $\varepsilon_{\mathrm{CF}}(\boldsymbol{k})$ in the x–y plane. However, the CF dispersion relation and the CF effective mass are actually unknown [20]. Taking this into account, we have integrated equation (8) considering two different CF effective-mass values, i.e., $m_{\mathrm{CF}}^* = 0.067m_0$ and $m_{\mathrm{CF}}^* = 0.5m_0$. The first value is equal to the GaAs effective-mass value in the conduction band. The dynamics has been studied in the absence of electric fields. Furthermore, at $t = 0$, the value of the oscillating magnetic field in each quantum well is unclear. Such an initial condition problem can be solved by considering all possible $\boldsymbol{B}_{L\,(R)}^*$-values at $t = 0$. Then, an averaged value for the peak position on the x-axis $\langle L\rangle$ can be obtained.

In figure 4, and in the double-quantum-well case, we have also plotted a CF trajectory in the $j = 1, 2$ cases considering charge-density oscillations between the two quantum wells. We can notice that the position of the focusing, L, on the x-axis is at a lower value than the right-hand constriction position ($x = 4$ μm). Such an effect is due to the existence of an oscillating effective magnetic field.

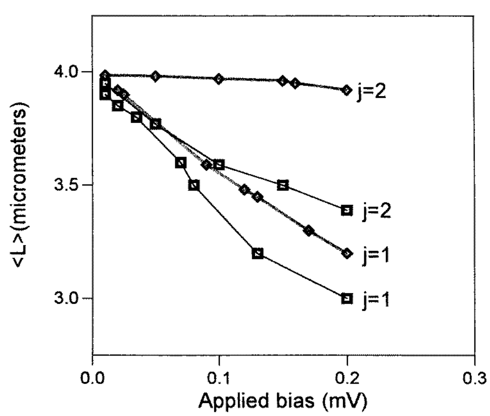

Figure 5. $\langle L\rangle$ versus the applied bias. Lines with diamonds: $m_{\mathrm{CF}}^* = 0.5m_0$. Lines with squares: $m_{\mathrm{CF}}^* = 0.067m_0$.

In figure 5, we show $\langle L\rangle$ versus the applied bias in the $j = 1$ and $j = 2$ cases. We can notice that the position of the focusing on the x-axis moves to lower values as we increase the applied bias. Such an effect can be easily explained as follows. If we increase the applied bias, the amplitude of the oscillating charge density is increased (figure 3). As a consequence, and from equation (7), the amplitude of the oscillating effective magnetic field is also increased. Then, the CF trajectories obtained are modified due to the existence of an oscillating effective magnetic field. At this point, we should remark that if $\langle L\rangle$ is decreased, the magnetic focusing position is shifted to a lower magnetic field value.

In such a case, a new kind of magnetoresistance measurement will be obtained in the double-quantum-well system. In the double-quantum-well case, we can expect to obtain two new effects in the magnetoresistance data. Firstly, and as we increase the applied bias, the different focusing peak positions will be shifted to lower magnetic field values. Secondly, we can also notice that we have obtained different values of $\langle L \rangle$ in the cases where $j = 1$ and $j = 2$ (figure 5). As a result, the new resistance measurements will not exhibit equidistant focusing peaks if B is increased.

In figure 5, it is also seen that the oscillating-field effect becomes important as we decrease the CF effective mass (i.e., as we increase $|\boldsymbol{v}|$). If the tunnelling frequency is much higher than the cyclotron frequency, then, in principle, the effect of the oscillating effective magnetic field on the CF trajectories can be neglected. In figure 5, we can also notice that, in the $j = 2$ case, the modification of the value of $\langle L \rangle$ obtained is not very large. In such a case, the dispersion of the L-values has been increased due to the CF reflections from flat gates.

In summary, in this work we have numerically integrated over space and time a nonlinear effective-mass Schrödinger equation for CFs in a double-quantum-well system. Many-body effects and an external applied bias have been considered in our model. It is found that the CF effective magnetic field in each quantum well displays oscillations as time progresses. This is in part related to the CF tunnelling process in the heterostructure growth direction. As a consequence, the possibility of there being a new kind of CF dynamics in the semiconductor layer plane of a double-quantum-well system has been demonstrated. In such a case, the magnetic focusing peak positions are shifted to lower effective magnetic field values. In principle, an experimental observation of such a process is possible.

Acknowledgment

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